Optimizing fleet size and scheduling of feeder transit services considering the influence of bike-sharing systems

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ABSTRACT

This study optimizes the fleet size and schedules of feeder buses that connect metro and residential areas in the context of bike-sharing systems. We propose hybrid operation modes that combine fixed and dynamic frequencies in a bimodal period in comparison with the conventional bus scheduling with constant service frequencies. The effect on endogenous demands from bike-sharing, which is another option for commuters, is considered. This study proposes a multi-objective model under one hybrid mode with morning fixed and evening demand-responsive (MFED) service to minimize the average passenger waiting time and maximize the operator profits. The constraints include vehicle capacity and passenger mode choices. Two algorithms are developed to solve the feeder bus planning problem of a metro station and three nearby communities in Chengdu, China (i.e., non-dominated sorting genetic algorithm-II [NSGA-II] and a customized multi-objective optimization algorithm based on Particle Swarm Optimization [MPSO]). Numerical results show that the proposed MPSO algorithm slightly outperforms NSGA-II in terms of solution quality and efficiency. We further compare two feeder transit operation modes (i.e., morning demand-responsive and evening fixed [MDEF] service and all fixed service) and find that the MFED outperforms MDEF and fixed operation mode in terms of system effectiveness. In addition, we confirm that the effect of bike-sharing systems cannot be neglected because passengers may change their travel mode while waiting for a feeder transit service. This study can provide useful policy implications and operational recommendations for government agencies and transit authorities to regulate the bike-sharing market for effectively addressing the first-and-last-mile issue.

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1. Introduction

In recent years, bike-sharing has become increasingly popular in many countries given its convenience and governmental advocacy. However, bus ridership has been slightly affected by bike-sharing because passengers opt to ride bicycles for commuting when transit service is inaccessible for short-distance travels (Campbell and Brakewood, 2017). The advent of bike-sharing systems holds great potential in solving the first-and-last-mile problem (Shaheen et al., 2010). However, this development triggers a series of new

safety issues, such as disorderly parking, particularly around the metro and bus stations, thereby occupying bicycle and motor vehicle lanes and partially or completely blocking sidewalks. Previous studies have confirmed that bike-sharing systems become minimally attractive under adverse weather conditions (e.g., snow, rain, and scorching heat) (Kim, 2018). The willingness of bike-sharing usage is expected to decline because the distance from residential areas to a metro station increases to a certain degree (Campbell and Brakewood, 2017). Under the abovementioned circumstances, where bike-sharing systems are infeasible, the feeder transit service remains advantageous because it provides easy access to nearby metro stations.

Generally, the transit planning process is comprised of five steps, namely, latent demand study, network and route design, timetable development, vehicle scheduling, and crew scheduling

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(Ceder and Wilson, 1986; Ceder et al., 2001; Yim and Ceder, 2006). The first one among these steps is the premise and foundation of the remaining ones which in turn affect passengers' travel choice behavior. For example, unreasonable timetable design may result in long waiting time, thus forcing people to select other travel modes, such as shared bikes or taxis.

The target potential users in the present study are mainly commuters. Thus, the feeder bus will operate for morning and evening periods. Then, the operators must decide on the required number of vehicles. Evidently, the fleet size is closely related to vehicle schedules and timetable. Several studies have focused on scheduling problem under the condition of a given fleet size (e.g., De Palma and Lindsey, 2001; Wang, 2017); in this case, the bus departure time is typically the decision variable, and the departure interval is flexible. By contrast, other studies have concentrated on the joint optimization of fleet size and bus schedules. This type of research is classified into two categories, that is, the frequency is fixed and dynamic. In former studies, the frequency was indirectly formulated by fleet size, so that there was only one independent variable rather than two (e.g., Wirasinghe, 1980; Kuah and Perl, 1989; Shrivastava and O'Mahony, 2006,2009; Furth and Wilson, 1981; Verbas and Mahmassani, 2015); in latter studies, the frequency was viewed dynamic as dial-a-ride (e.g., Cordeau and Laporte, 2007; Lu et al., 2016) and first or last-mile (e.g., Wang and Odoni, 2014; Shen et al., 2018) problems, which belong to demand-responsive transit (DRT).

However, few studies on the feeder transit service have addressed the joint optimization of fleet size and bus schedules that integrate morning and evening periods. A cohesive solution for the entire transit planning problem is difficult to achieve by sequential approaches to each sub-problem (Ibarra-Rojas et al., 2015). For example, the morning and evening travel demands have five and seven optimal fleets, respectively. If the entire travel demand over the two periods is considered, then the optimal fleet is six. Thus, we must concurrently consider the fleet size of the feeder transit service during hybrid service periods and operation modes. Most feeder transit systems use two typical operation modes, namely, fixed and demand-response schedules (Li and Quadrifoglio, 2010). The traditional feeder transit system with a fixed schedule is increasingly cost efficient given the predetermined schedule. However, this system lacks flexibility in satisfying individual passengers' travel requirements. By contrast, the demand-responsive feeder transit system can provide door-to-door service with a dynamic schedule from a metro station to an individual's home at the cost of considerable operational resources. Existing studies have determined that the demand-responsive feeder transit system is more favorable than the fixed feeder system during the evening, night, or early morning hours when demand is low (Daganzo, 1984; Koffman, 2004). Li and Quadrifoglio (2010) quantitatively verified that the demand-responsive feeder transit service must be implemented during peak hours when the demand falls between 10 and 50 passengers/mile^2/h. However, the study of these authors was based on simulation and assumption that passenger demand follows Poisson distribution with no passenger leaving due to excessive waiting time. Under the advent of bike-sharing systems (especially dockless shared bikes), passengers can take bicycles from their home places to the metro station. Therefore, the loss of passenger demand caused by bike-sharing must be considered to decide on whether fixed or demand-responsive feeder transit service mode must be implemented.

The present study optimizes the fleet size and scheduling for feeder buses in bimodal periods to address the aforementioned issues considering the effect of bike-sharing systems on endogenous demand. The research objective of this study is to explore the optimal operational strategy in a multi-modal transit network for

transit authorities, including how many bus fleets need to be purchased in advance and how to dispatch the vehicles efficiently. A hybrid operating mode that combines fixed frequency and dynamic schedules, rather than the conventional morning fixed and evening fixed service (MFEF), during the bimodal periods is adopted. A multi-objective model is developed to acquire coordinative optimization between operator and passenger waiting costs. This will help transit authorities make insightful decisions considering both operating cost and service quality comprehensively, and avoid resources to be wasted for efficient feeder bus service. We formulate the problems in the morning and evening and connect them with the same fleet size. Different from single-objective optimization problem (e.g., Wang et al., 2017a,b; Changxi et al., 2018a,b), multi-objective optimization problem is more complicated in algorithm design. A customized Particle Swarm Optimization (PSO) algorithm is then designed to obtain Pareto solutions for solving the problem. Moreover, we compare with non-dominated sorting genetic algorithm-II (NSGA-II), which is proposed by Deb, Pratap, Agarwal, & Meyarivan in 2000 and has been extensively used to solve multi-objective problems (Wang and Liu, 2015, Rabbani et al., 2017; Rashidnejad et al., 2018; Yong et al., 2018).

The primary contributions of this study are presented as follows: (1) The fleet size is considered a decision variable under a hybrid operation mode with a dynamic schedule. An improved multi-objective optimization algorithm based on PSO (MPSO) algorithm is designed to solve this complex multi-objective problem for balancing operation costs and service quality. (2) The effect of bike-sharing systems on consumers' travel choice is incorporated in the proposed model, which indirectly affects the endogenous passenger demand and feeder bus scheduling. (3) The morning and evening period service modes are simultaneously studied to decide on the optimal fleet and schedules for transit operators. We perform an in-depth comparable framework to examine the service modes between fixed and demand-responsive feeder transit services on the basis of empirical data in Chengdu, China.

The remainder of this paper is organized as follows: Section 2 reviews the literature. Section 3 introduces the problem in detail and states our assumptions. Section 4 formulates the multi-objective model under a hybrid operation mode. Section 5 describes the MPSO algorithm for solving the problem. Section 6 demonstrates the algorithm comparison, sensitivity analysis, and service mode comparison through numerical analysis. Section 7 presents the conclusions and future research area.

2. Literature review

The creation of timetables on fixed routes has a close relation to the dispatching policy problem, which has been extensively studied in the literature. Newell (1971) and De Palma and Lindsey (2001) investigated this problem in an idealized transit system. Specifically, Newell (1971) assumed the passenger arrival rate as a given smooth function of time and studied the dispatching schedule for a transportation route in two situations, that is, whether to consider vehicle capacity or not. This work aims to minimize the total passenger waiting time. De Palma and Lindsey (2001) analyzed the optimal timetable for a given number of vehicles between two nodes on a single line considering riders' preferred travel times that were assumed to be uniformly distributed in the population for minimizing the total cost of schedule delay. These authors ignored vehicle capacity constraints. Adamski (1998) developed a dynamic optimal dispatching control of vehicles using a simulation model to increase the reliability of bus operations in terms of on-time performance. Furth and Wilson (1981) developed a model to set frequencies by allocating available buses to maximize the net social benefit subject to constraints in terms of

total subsidy, fleet size, and levels of vehicle loading. Verbas and Mahmassani (2013) extended the model presented by Furth and Wilson (1981) by introducing two formulations. The first formulation was to maximize the total wait time savings and number of riders under the constraints of bus loading, policy headway, fleet, and budget. The second formulation was to minimize the net cost under the same constraints with the first formulation. Through a numerical test, this work found that the increment of fleet size may cause the reduction of operational cost.

Among the studies applied to real cases, Ceder (1986) provided alternative methods for creating bus timetables using passenger load data and produced a set of computer programs to decide on bus departure times for the case of evenly and unevenly spaced headways in which the average loads are uniform. Hadas and Shnaiderman (2012) presented an approach for frequency setting using statistical distributions of passenger demand and travel time developed from automatic passenger counter and automatic vehicle location data. This work addressed the minimization of the total cost incurred with frequency or vehicle capacity on the basis of empty seats and unserved demand. Verbas and Mahmassani (2015) used a bi-level programming model to solve the frequency-setting problem and applied this model to numerous instances. However, the total transit demand was assumed to be fixed in their work. Martínez et al. (2014) considered the transit assignment decisions and proposed a model to jointly decide on the demand and frequency on each line. The proposed metaheuristic is applied to a large-scale realistic network. Cortés et al. (2011) developed a model by combining short turning and deadheading strategies for a single line and found that the integrated strategy can be justified in cases where unbalances are evident between and within areas. An empirical study in Santiago was undertaken in their numerical applications.

Although high-quality solutions can be obtained for each sub-problem (e.g., timetabling and vehicle scheduling) of the transit network planning, a cohesive solution for the entire transit planning problem is difficult to formulate through sequential approaches. Thus, recent studies have focused on integrating two or more sub-problems of transit network planning. For example, Van den Heuvel et al. (2008), Ceder (2011), Petersen et al. (2013), and Ibarra-Rojas et al. (2014) investigated the integration of timetabling and vehicle scheduling problems.

Research on feeder bus network scheduling problem or last-mile problem also have been extensively investigated. Yim and Ceder (2006) documented the survey results of Castro Valley on the smart feeder/shuttle bus service and proposed 10 routing and scheduling strategies. Most related papers have solved the feeder bus scheduling problem considering the schedules of a trunk line or node. For example, Sivakumaran et al. (2012) coordinated the schedules of the feeder line with those of the trunk line in an idealized system and concluded that user and operator costs diminish when the frequencies of feeder and trunk lines are jointly established. Xiong et al. (2015) optimized a synchronized timetable for community shuttles connected to a metro considering vehicle capacity and fleet size constraints. These authors proposed a heuristic algorithm of shifting departure times to solve the model. Wang (2017) presented a mixed-integer programming model to determine the schedules for a multi-vehicle fleet for a last-mile problem, thus minimizing passenger riding and waiting times. Chandra et al. (2013) analyzed the accessibility effects to a major transit line/transfer stop using two most common feeder transit services, namely, fixed route transit and DRT. Li and Quadrifoglio (2010) developed analytical and simulation models to help decision-makers to select between a fixed-route and a demand-responsive operating policy and when and whether to switch from one to the other during the day. Pan et al. (2014) presented a

mixed integer linear programming model for a flexible feeder transit system to design the routing plans and service area given the fleet size. Ceder (2013) applied a designed simulation model to investigate 10 routing strategies with all the different combinations of fixed/flexible schedules, shortcut and/or short-turn concepts and so on.

Most studies, such as Hsu et al. (2018), Zhang et al. (2015), Zhang et al. (2019), and Haider et al. (2018), involving bike-sharing have focused on bike-sharing itself, including the evaluation of its service quality, characteristic analysis, electric fence planning, inventory, and rebalancing. For example, Hsu et al. (2018) proposed a model that considers information uncertainty and various criteria for evaluating and improving the service quality of public bike-sharing systems. Several remaining studies, such as Campbell and Brakewood (2017), have addressed the interaction between bike-sharing and public transit. However, these studies have only analyzed the system effect on travel demand and have not further considered bus planning under such influence.

In summary, most existing literature has focused on the joint optimization of traditional bus schedule and fleet but has lacked research on feeder bus joint optimization. In addition, morning and evening peaks have not been considered jointly in existing literature, and studies on the manner by which service mode is determined by flexible or fixed mode have been rare. Therefore, the methods for optimizing feeder bus scheduling in a competitive way on dynamic demand must be further explored. In the present study, we investigate the integration of feeder bus dynamic scheduling and fleet management problem in contrast to a traditional fixed frequency one on multiple lines. In addition, the effect of bike-sharing systems, as a competitive way, is considered. This endeavor is rarely referred to in previous literature.

3. Problem description

An area with a metro station in the center within a radius of 5 km is considered and depicted in Fig. 1. People living in this area routinely travel between their home places and the metro station for commuting. In general, we can set up the stops on one side of feeder buses by mining bike-sharing trajectory and transit smart data through several clustering methods (Bordagaray et al., 2016). The stop on the other side is evidently the metro station. Here, the routes are assumed to be predetermined. The problem is the number of vehicles that are required for the operator and type of vehicle scheduling considering the influence of bike-sharing systems.

3.1. Consumer's choice behavior

Shared bikes and feeder buses are the two main choices for rail transit riders to reach the metro service. A bike-sharing scheme is cheap and exhibits high reliability. Thus, if a person knows his/her cycling speed, then the time he/she takes to reach the metro station is nearly certain. If taking feeder buses, then passengers must walk to the bus stop first, then wait for a bus to arrive, and finally board the bus. In this process, the waiting time generally depends on passengers' arrival time with the varying in-vehicle times caused by traffic congestion. This condition indicates that the reliability is worse in feeder buses than that in a shared bike. However, a bus evidently runs faster under ideal situations and is also labor-saving for long-distance travels, especially during the rainy and snowy seasons.

Fundamentally, passengers will not take feeder buses if the trip cost (which includes walking, waiting, and in-vehicle) is much higher by bus than by shared bikes (cycling cost), except for special groups, such as the elderly or disabled. Moreover, if the waiting
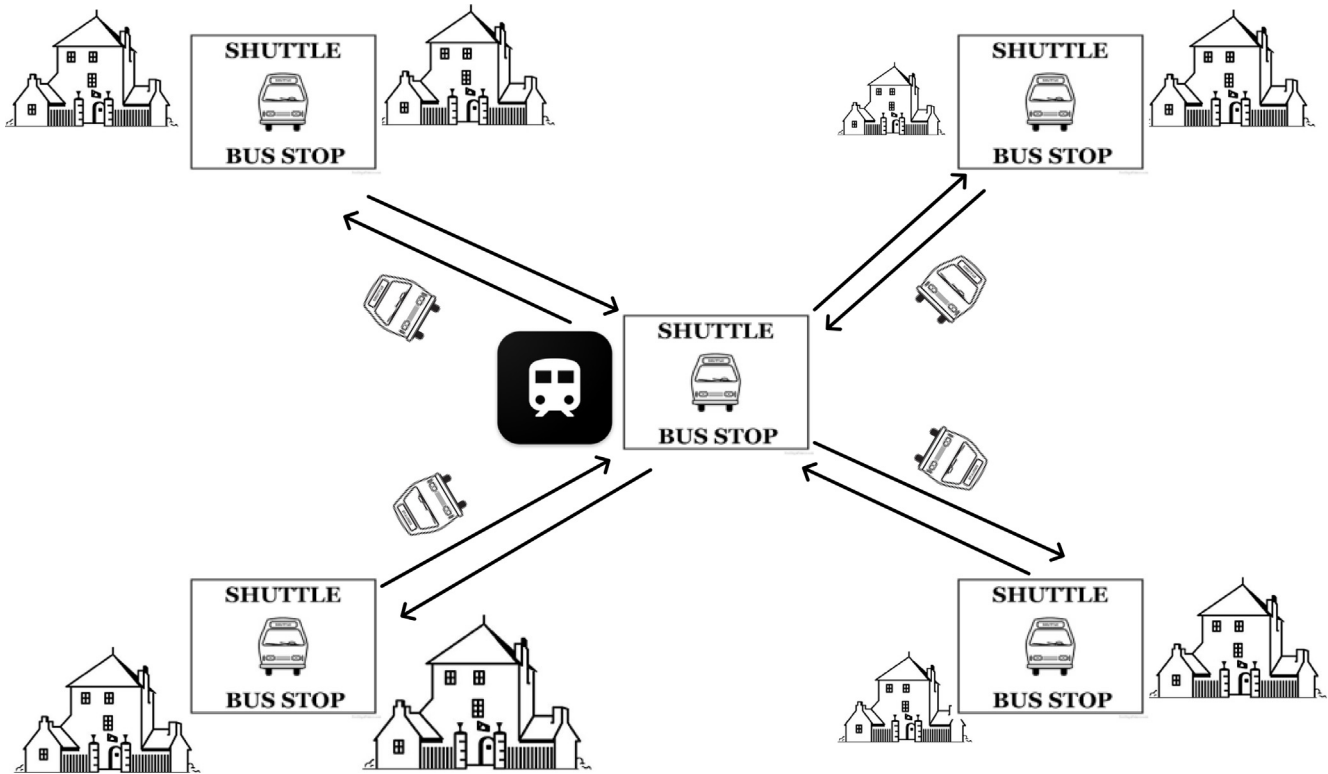
Fig. 1. Diagram of the feeder transit service problem.

time exceeds the acceptable value, then passengers may temporarily reconsider taking a taxi or riding a bike. Thus, implementary results may not be convincing without considering the effect of bike-sharing on feeder buses. We consider the effects from two points on the basis of these analyses. First, the trip time by feeder buses cannot exceed that by shared bikes. Second, passengers can switch to cycling or taxi while waiting for the feeder buses.

3.2. Hybrid operation modes

Generally, two combinations, namely, morning fixed and evening demand-responsive (MFED) service and morning demand-responsive and evening fixed (MDEF) service, exist in each route. The bus headway is equal within the fixed frequency mode, and the average waiting time is half the headway (Newell, 1971). The operator dispatches buses in accordance with varying demands and available vehicles within the dynamic scheduling mode. In this mode, passengers must provide their information through a smartphone app or internet website for helping the operator make plans in advance. Specifically, if the demand-responsive transit service is provided in the evening rush hour (in this case the feeder bus mainly serves the alighting commuters exiting from each metro station for a short bridging trip), a commuter need to submit his/her estimated arrival times at each metro station and the nearest bus stop to his/her homeplace (we do not allow passengers to input home addresses for privacy protection). For example, a passenger boards a train at 5:00pm and requires 20 min to arrive at his/her destination metro station (i.e., the studied metro station), he/she can submit the arrival time as 5:20pm and the bus stop that is most adjacent to his/her homeplace.

The modeling methods of the two combinations are essentially the same. Thus, we illustrate the MFED service in subsequent chapters. The performance of each service mode is closely related to the actual travel demand distribution. In the morning, passenger departure times from homes depend on their expected working times (mostly before 9 a.m. in China), and thus the morning passenger demand is concentrated. In the evening, passengers egress in batches from each metro station, thereby relying on the running interval of each train. In contrast to morning commuting travels, passengers do not have rigorous time windows to go home. Thus, the evening demand is generally dispersed. This could be validated by smart card transaction data collected from October 1, 2018 to October 31, 2018. We aggregated the total number of boarding trips by each minute in Fig. 2, and found that the boarding time were normally distributed in the morning rush hour, where the peak can be observed at 7:50 on weekdays. In the evening rush hour, there were several peaks in the boarding time. The three peaks appeared at 17:15, 17:45 and 18:15, respectively, which lag 15 min behind the typical off-duty times (17:00, 17:30 and 18:00). We initially adopt the conclusion of Li and Quadrifoglio (2010) on the basis of these characteristics and assume fixed and dynamic schedules in the morning and evening, correspondingly. The reverse service mode will be discussed in Section 6.

Notably, feeder buses also carry passengers on the return journey, but as previously mentioned, the main passengers for feeder buses are commuters. Thus, we neglect passengers on return journeys in the model because the number of returning passengers during peak hours is relatively small.

3.3. Operators' strategy

An operator must rationally determine the fleet size on the basis of the known demand. The assumption is that the morning period is $[t_s, t_e]$, and the evening period is $[T_s, T_e]$. Here, the number of running vehicles in the morning is the same as that in the evening. The vehicle capacity is uniform and denoted by c. The bus price is p
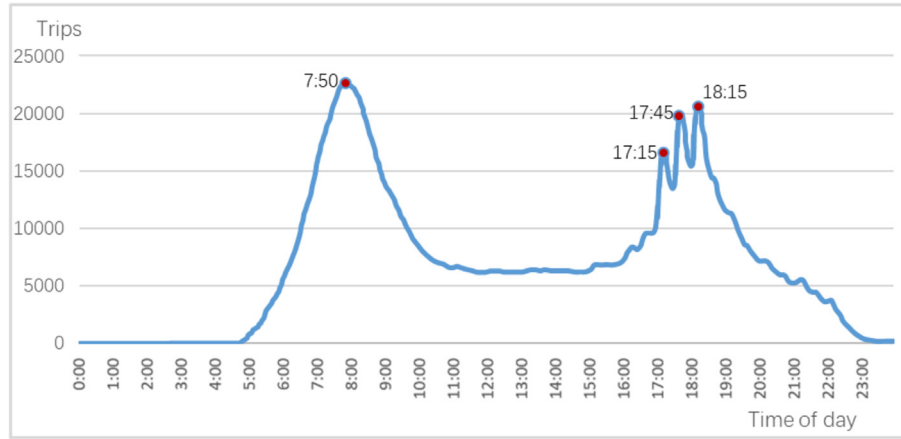
Fig. 2. Distribution of boarding time for Beijing public transport.

($) per passenger. Then, the operator's profit is the fare revenue. Furthermore, the expenditures are the vehicle purchase and operation expenses, which are proportional to fleet size and vehicle capacity. Jansson (1980, 1984) emphasized that the purchase cost and use of each vehicle demonstrate a linear dependency on bus capacity. In the present study, we denote the corresponding exogenous unit cost by C_B and C_K. Then, the expenditure can be computed as fleet size times each vehicle cost.

The following main assumptions are presented in this study:

(1) Feeder bus stops and routes are pre-specified.
(2) The number of running vehicles in the morning is the same as that in the evening (denoted by m). The vehicle capacity is c. The morning period is $[t_s, t_e]$, and the evening period is $[T_s, T_e]$.
(3) Passengers must submit to the bus operator the information on their arrival time and destinations in advance.

4. Multi-objective optimization model of the MFED service

In this section, we present a multi-objective model for solving the optimal fleet size and scheduling problems of a feeder bus. The variables are defined in Table 1.

4.1. Objective function

As stated previously, the running and layover times of vehicles for each feeder bus line are assumed to be fixed. Then, in-vehicle riding time is unnecessary when optimizing passengers' travel cost. Therefore, one objective function of the model is defined as minimizing the weighted sum of the average waiting time spent by all passengers (i.e., average waiting times in the morning and evening).

$$\min \ W\left(m_j, t_j^i\right) = \gamma_m \frac{\sum_j \frac{1}{2}\left(\sum_i x_{mj}^{t_j^i}\right)\Delta t_j}{\sum_j \sum_i x_{mj}^{t_j^i}} + \gamma_a \frac{\sum_t \sum_j P_{aj}^t}{\sum_t \sum_j y_{aj}^t}. \tag{1}$$

In this formula, γ_m and γ_a represent an individual passenger's sensitivity level of waiting in the morning and evening, respectively. On the one hand, we aim to minimize the waiting time, rather than maximize the number of served passengers, to improve the service quality of public transit for balancing with bike-sharing service, rather than replacing it. On the other hand, the total waiting time for all passengers cannot adequately reflect the level

of service for feeder buses because the number of served passengers varies with the changes in a timetable.

The other objective function of the model is defined as minimizing the operator's cost (i.e., expenses minus fare profits), which is detailed in Section 3.3.

$$\min Z\left(m_j, y_{aj}^t, h_j^t\right) = mRT(C_B + C_K c) - p\left(\sum_j \sum_i x_{mj}^{t_j^i}\right.$$
$$\left. + \sum_t \sum_j y_{aj}^t\right). \tag{2}$$

4.2. Constraints

The problem presented the following constraints:

(a) **Fleet size constraints**: In Fig. 1, the bus stop near residential areas is marked by $j (j = 1, \dots, n)$, and the stop near the metro station is denoted by ST. In the morning, the bus headway between bus stop j and stop ST is defined as Δt_j. The running and layover times are supposed to be t_{rj} and t_{lj}, correspondingly. Then, the corresponding fleet is computed using (3), and the fleet size m is equal to the sum of vehicles running on each line.

$$m_j = \frac{2\left(t_{rj} + t_{lj}\right)}{\Delta t_j}, \quad \forall j, \tag{3}$$

$$m = \sum_j m_j. \tag{4}$$

(b) **Trip time constraints**: If the average walking time for passengers living around stop j to reach the nearest bus stop is $\overline{t_{wj}}$, then the riding time will be $\overline{TB_j}$ if passengers select shared bikes for commuting to the metro station. Here, we assume that the longest waiting time is Δt_j. Specifically, a passenger arrives at the bus stop when the previous bus has just left and must board the next bus. A passenger's trip time using feeder buses is set to be no longer than the riding time to be competitive with bike-sharing systems.

Table 1
Notation for the model.

Parameters	
$\overline{t_{wj}}$:	average walking time for passengers from their origins to stop j in the morning
Δt_j:	headway of vehicles at stop j in the morning
t_{rj}:	running time of vehicles between stop j and the metro station
$\overline{TB_j}$:	average riding time between stop j and the metro station
t_{lj}:	layover time of vehicles between stop j and the metro
t_{sj}:	round-trip time between stop j and the metro
c:	vehicle capacity
RT:	total operating time
$n_{mj}^{t_j^{i-1} \to t_j^i}$:	number of passengers at stop j whose arrival time is between t_j^{i-1} and t_j^i in the morning
n_{aj}^t:	number of passengers with destinations near stop j arriving at time t in the evening
Decision variables	
m_j:	number of vehicles operating between stop j and the metro station in the morning
t_j^i:	ith vehicle's arrival time at stop j in the morning
y_{aj}^t:	number of passengers served by the vehicle with destination near stop j at time t in the evening
h_j^t:	number of vehicles with destination near stop j initiated at time t in the evening
Intermediate variables	
$x_{mj}^{t_j^i}$:	number of passengers at stop j served by the vehicle arriving at t_j^i in the morning
$P_{mj}^{t_j^i}$:	number of unserved passengers waiting at stop j at the end of time t_j^i in the morning
m:	total number of vehicles
P_{aj}^t:	number of unserved passengers with destinations near stop j at the end of time t in the evening
v^t:	number of available vehicles at the metro station at the end of time t in the evening

$$\overline{t_{wj}} + \Delta t_j + t_{rj} \leq \overline{TB_j}, \quad \forall j. \tag{5}$$

(c) **Passenger flow constraints**: For stop j, in the morning period, the cumulative number of passengers that arrive between the time vehicle i leaves the stop, and the time when vehicle $i+1$ arrives at the stop is assumed to be $n_{mj}^{t_j^{i-1} \to t_j^i}$. The number of passengers boarding vehicle i is denoted by $x_{mj}^{t_j^i}$. Expressions (6)–(7) denote the passenger flow balance in which α is the loss rate of passengers during the waiting period, and a portion of passengers may shift to shared bikes. In the evening, passengers exit from the metro station and have different destinations. Here, we discretize the time into a 1-min interval, as presented by Hai Wang (2017). Then, the passenger flow must satisfy Equations (8) and (9), in which β is the loss rate of passengers during the waiting period, at the end of time t for the destination stop j.

$$P_{mj}^{t_j^1} = n_{mj}^{t_j^0 \to t_j^1} - x_{mj}^{t_j^1}, \quad \forall j, \tag{6}$$

$$P_{mj}^{t_j^i} = \alpha P_{mj}^{t_j^{i-1}} + n_{mj}^{t_j^{i-1} \to t_j^i} - x_{mj}^{t_j^i}, \forall j, \, i \geq 1 \tag{7}$$

$$P_{aj}^{T_s} = n_{aj}^{T_s} - y_{aj}^{T_s}, \quad \forall j \tag{8}$$

$$P_{aj}^t = \beta P_{aj}^{t-1} + n_{aj}^t - y_{aj}^t, \forall j, t \in [T_s, T_e] \tag{9}$$

(d) **Service capacity constraints**: The number of served passengers per vehicle must not exceed the vehicle capacity.

$$x_{mj}^{t_j^i} \leq c, \forall j, \, i \tag{10}$$

$$y_{aj}^t \leq c, \forall j, \, t \tag{11}$$

(e) **Vehicle flow constraints**: Equations (12)–(13) define the number of available vehicles in the transit process in the evening.

$$v^{T_s} = m - \sum_j h_j^{T_s}, \tag{12}$$

$$v^t = v^{t-1} + \sum_j h_j^{t-t_{sj}} - \sum_j h_j^t, \quad \forall t \in [T_s, T_e]. \tag{13}$$

(f) **Operating time constraint**: The total operating time is equal to the sum of the morning and evening operating times.

$$RT = (t_e - t_s) + (T_e - T_s). \tag{14}$$

(g) **Other basic constraints**: Equations (15)–(17) regulate the ranges of several decision variables.

$$t_s \leq t_j^i \leq t_e, \forall j, \, i \tag{15}$$

$$v^t \geq 0, \forall t \in [T_s, T_e] \tag{16}$$

$$P_{aj}^t > 0, \forall j, t \in [T_s, T_e] \tag{17}$$

The bi-objective mixed integer programming model is NP-hard because a portion of the model can be denoted as the $(P, \text{cap}\boldsymbol{c} | S, P_{aj} | \text{Graph} | \sum C_j)$ problem, as presented by de Paepe et al. (2004); this problem^{j}has been verified to be NP-hard. Therefore, heuristics are required to obtain the non-inferior solutions for a large-scale instance of our model.

5. MPSO

In this section, a new algorithm (i.e., PSO) combined with a heuristic algorithm of shifting schedules is proposed to solve the problem. The input of the algorithm is passenger's demand, transit round-trip time for each route, riding time, and other parameters. The output is fleet size and timetable.

The PSO algorithm was first proposed by Kenndy and Eberhart in 2002. In this algorithm, each particle represents a potential solution to the problem and corresponds to a fitness value computed by the fitness function. The particle velocity, which is dynamically adjusted with the moving experience of itself and other particles, determines the moving direction and distance, thereby optimizing the goal in a feasible solution space. Goldberg et al. (1988) proposed to solve the multi-objective optimization problem by combining Pareto theory in economics with an evolutionary algorithm. Thus, we receive a non-inferior solution set. In practical application, the decision-maker selected one of the non-inferior solutions as the final solution in accordance with their preference.

Subsequently, we will introduce the manner by which to use the new algorithm for solving our problem in detail.

5.1. Initial feasible solution

In this study, the key point is to determine the bus frequency for each line in the morning and schedules of different bus stops in the evening. Then, iterations are continuously conducted by computing the objective value, and the optimal solution can be finally achieved. Notably, multiple scheduling schemes can be generated for each fleet value. Thus, the method for creating a feasible solution is first generated by the bus headway for every stop in the morning according to Constraints (5)–(15). Then, the fleet size can be determined using Constraints (3)–(4). Finally, the timetable in the evening can be generated on the basis of Constraints (12)–(13) and (16)–(17) (see Fig. 3).

The fitness value of the particle is computed by an objective function. Before calculating the average waiting time of all served passengers and revenue of the operator, the real-time demand of passengers in the morning and evening must be updated, thereby indicating that the demand varies with the bus schedule and is related to bus capacity (see Fig. 4).

5.2. Regeneration of solutions

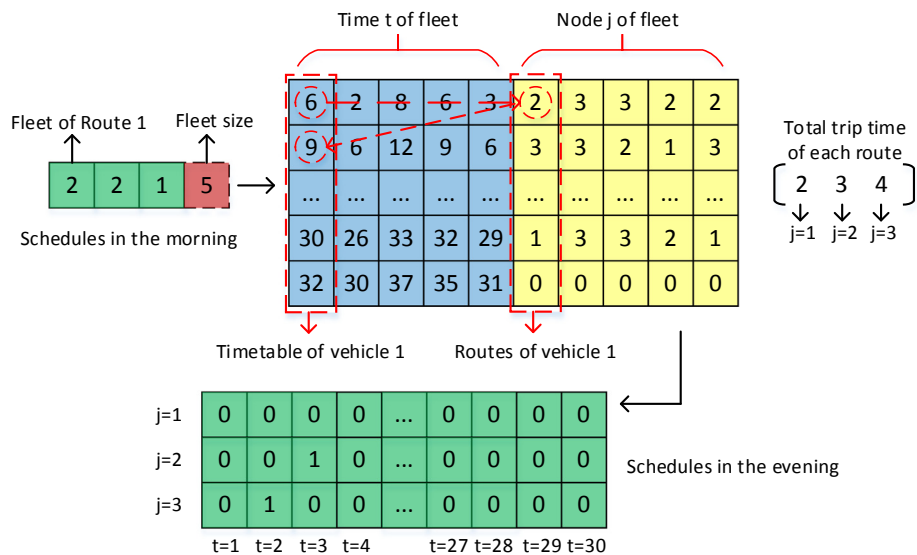The speed and position of the current particle are updated on
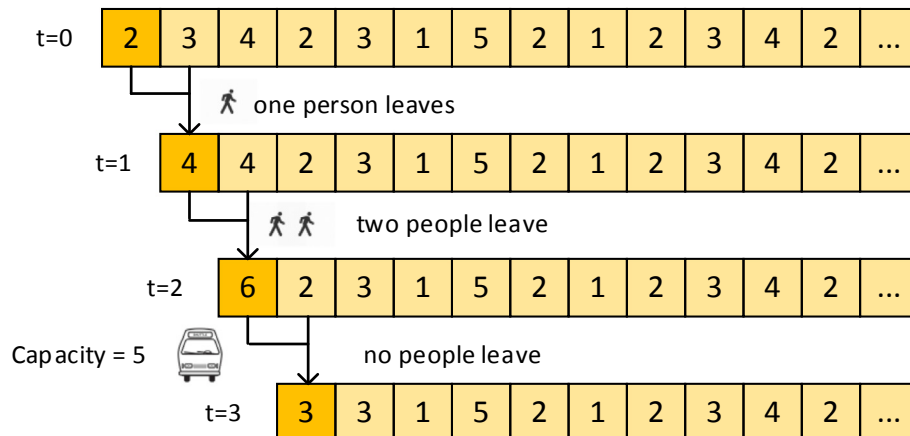

Fig. 3. Example of a solution structure.

Fig. 4. Example of updating demand.

the basis of an individual's and overall optimal particle, which is randomly selected from the non-inferior solutions. The formula for updating particles is expressed as follows:

$$V^{k+1} = \omega V^k + c_1 r_1 \left(P_{id}^k - X^k \right) + c_2 r_2 \left(P_{gd}^k - X^k \right), \tag{18}$$

$$X^{k+1} = X^k + V^{k+1}, \tag{19}$$

where ω is the inertial weight; r_1 and r_2 are the random numbers distributed in [0,1]; k is the number of current iterations; P_{id}^k denotes the individual's optimal particle position; P_{gd}^k is the global optimal particle position; c_1 and c_2 are constants; V is the particle velocity; and X is the particle position.

5.3. Process of the proposed MPSO algorithm

```
BEGIN
  Initialization;
    Randomly generate the bus headway for each stop in the morning
      Repeat
  If all population are computed, then
      Stop.
  Else
    Repeat
  If all stops are enumerated, then
      Stop.
  Else

    Compute the total number and waiting time of served passengers for stop  J̇   in the
    morning
    Repeat
    If all vehicles of the ith population are enumerated, then
      Stop.
    Else
      Randomly generate the schedules of the kth vehicle of the ith population in the evening
    Repeat
    If all stops are enumerated, then
      Stop.
    Else
      Obtain the vehicle schedules for stop J̇ _ in the evening
    Compute the number of unserved passengers at the end of time t and the total number of
    served passengers
      Then, compute the objective values, which are the total average waiting time and
      operator's profits
    Repeat
      If the termination condition is satisfied, then
        Stop.
      Else
        Update the weights
        Select a particle from the non-inferior solutions as the global optimal solution
        Update the population, including the velocity and position
        Recalculate the objective values, which are the total average waiting time and operator's
        profits
        Update the non-inferior solution set
  End
```

---

6. Numerical analysis

In this section, we use empirical examples to solve and verify

our model. The transit smart and bike-sharing trajectory data within 2 km of Shuangqiao station on Line 4 of the Chengdu metro system in China are used to identify the travel demand. Fig. 5 illustrates the location of three residential communities and the metro station, and Fig. 6 plots the existing bus routes from these communities to the metro station. Apparently, these routes that connect communities and metro stations are not well-organized, thereby leading to an average trip time of 30 min to the metro station from each residential community. However, the average trip time when taking a taxi is approximately 10 min. We suppose that three feeder bus lines follow the same routes that the taxi travels. The operation time is half an hour in the morning (7:00 a.m. to 7:30 a.m.) and half an hour (6:30 p.m. to 7:00 p.m.) in the evening.

6.1. Algorithm comparison

We implement and test NSGA-II to compare with the proposed MPSO for exploring the solution quality and efficiency among different algorithms. Notably, the methods of chromosome representation, initial population, and fitness evaluation using NSGA-II is the same as using the proposed MPSO because both of these

Fig. 5. Geographical distribution of the communities and metro station.

approaches are used to solve the same model. The parameters of the test problem are presented in Table 2.

The operation times are half an hour in the morning and half an hour in the evening, during which the demand is listed in Table 3. Here, we use weekly average travel data. In both algorithms, the number of population and iterations are set to 100 and 200, respectively. The results are summarized in Table 4.

In Table 4, the number of Pareto front solutions is set to 8 in NSGA-II. We select the approximate solutions from the solution set of the proposed MPSO to compare the Pareto solutions computed through the two algorithms. The results obtained from both algorithms support the fact that increasing the operator's cost leads to a reduction of a passenger's average waiting time, although the specific values are slightly different. The objective values are different because the detailed schedules in the operation period vary despite having the same fleet size.

Two main criteria, namely, solution quality and computational cost, are typically used to compare and evaluate the algorithm performance. In terms of the solution quality, two measures, namely, diversity and convergence, are commonly used. The former implies the degree of dispersion of the non-dominated solutions, while the latter indicates the closeness of the Pareto non-dominated front solutions to the Pareto optimal front. Here, we use mean ideal distance (MID) to evaluate the convergence of two algorithms using Equation (20).

$$MID = \frac{\sum_{i=1}^{n} N_i}{n}, \tag{20}$$

where N_i denotes the distance from an ideal point (0,0) for each solution i. Then, the normalized values of the MID for NSGA-II and proposed MPSO are 0.1982 and 0.1897, correspondingly. The results show a small difference between the MID of the two algorithms, thereby indicating that determining the one that performs well in terms of convergence is difficult. However, for the diversity of solutions, the fleet sizes are more diverse and uniformly distributed in the proposed MPSO than in NSGA-II.

In addition, the elapsed time is slightly different. Specifically, the elapsed time of NSGA-II for the test problem with four feeder bus stops is 7032.53 s, whereas the proposed MPSO solves the problem in 4691.97 s. Therefore, the proposed MPSO is more efficient than NSGA-II.

6.2. Sensitivity analysis

In this section, we perform a sensitivity analysis of the parameters involved in the model. The results of the proposed model under different parameter settings are tested on the basis of the actual travel data. The testing parameters include the loss rate of passengers during the waiting period in the morning and evening and the vehicle capacity (demonstrated in Figs. 7 and 8).

Fig. 7 exhibits the sensitivity of the objective function to different loss rates. Three cases, namely, $a = 1$, $b = 1$ (no passengers leave during the average waiting time); $a = 0.95$, $b = 1$ (several passengers leave during the average waiting time in the morning); and $a = 0.9$, $b = 0.95$ (several passengers leave during the average waiting time in the morning and evening), are tested and compared. The results show that the operator's cost significantly
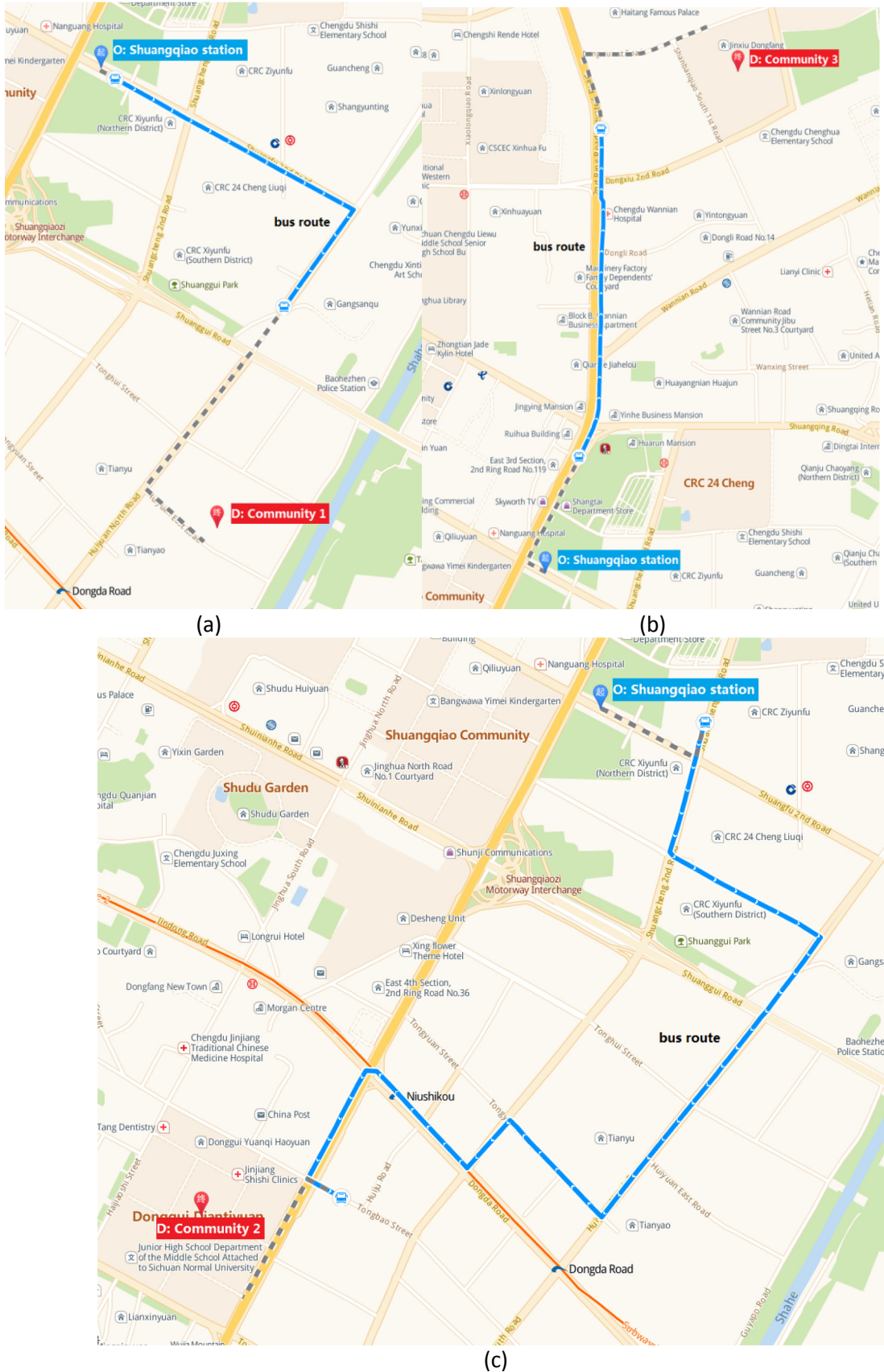
Fig. 6. Existing bus routes from the metro station to residential communities: (a) commutiny 1; (b) community 3; (c) community 2.

Table 2
Parameters for numerical simulation.

Parameter	t_{r1}	t_{r2}	t_{r3}	t_l	$\overline{t_{w1}}$	$\overline{t_{w2}}$	$\overline{t_{w3}}$
Value (units)	5 min	8 min	10 min	2 min	4 min	5 min	6 min
Parameter	c	p	γ_m	γ_a	a	b	RT
Value (units)	15 seats	$ 0.45	0.7	0.3	1	1	1 h
Parameter	$\overline{TB_1}$	$\overline{TB_2}$	$\overline{TB_3}$	C_B		C_K	
Value (units)	15 min	20 min	25 min	$ 7.48/h		$ 0.75/h-seat	

Fig. 8 illustrates the sensitivity of the objective function to different vehicle capacities. Similarly, we analyze the cases where the vehicle capacities are 10, 12, and 15. A small vehicle capacity indicates an improved solution quality when the passenger waiting time is between 2 and 3 min.

6.3. Mode comparison

6.3.1. Comparison with the MDEF mode

Table 3
Demand in the operation period.

| Time (min) | Passenger demand (passengers) | | | | | |
| | Morning operation period (O: bus stops; D: metro station) | | | Evening operation period (O: metro station; D: bus stops) | | |
	Bus stop 1	Bus stop 2	Bus stop 3	Bus stop 1	Bus stop 2	Bus stop 3
$t = 1$	4	1	2	4	8	6
$t = 2$	2	3	3	0	0	0
$t = 3$	1	1	4	0	0	0
$t = 4$	3	3	2	8	4	8
$t = 5$	3	1	1	0	0	0
$t = 6$	4	2	4	0	0	0
$t = 7$	2	2	1	6	7	5
$t = 8$	3	3	1	0	0	0
$t = 9$	4	1	1	0	0	0
$t = 10$	1	2	2	5	5	10
$t = 11$	2	3	3	0	0	0
$t = 12$	2	2	2	0	0	0
$t = 13$	1	2	3	5	9	9
$t = 14$	3	2	1	0	0	0
$t = 15$	2	2	3	0	0	0
$t = 16$	2	2	3	8	6	5
$t = 17$	4	4	1	0	0	0
$t = 18$	1	2	2	0	0	0
$t = 19$	5	1	4	9	6	6
$t = 20$	3	4	3	0	0	0
$t = 21$	5	1	5	0	0	0
$t = 22$	1	1	1	5	9	7
$t = 23$	3	2	2	0	0	0
$t = 24$	1	1	2	0	0	0
$t = 25$	3	4	2	7	8	9
$t = 26$	2	2	4	0	0	0
$t = 27$	1	1	1	0	0	0
$t = 28$	1	4	3	8	10	7
$t = 29$	4	1	3	0	0	0
$t = 30$	2	3	2	0	0	0

Table 4
Comparative results of NSGA-II and the proposed MPSO.

| Number of solutions | NSGA-II | | | Proposed MPSO | | |
	Operator's cost ($)	Average waiting time (min)	Fleet size	Operator's cost ($)	Average waiting time (min)	Fleet size
1	46.08	9.314	5	57.30	8.614	5
2	74.95	7.224	6	78.54	6.829	6
3	105.62	5.594	7	139.73	5.103	7
4	142.57	4.985	8	151.55	4.247	8
5	279.16	3.091	12	177.28	4.116	9
6	299.80	2.866	12	207.05	3.255	10
7	305.34	2.573	13	244.90	3.195	11
8	338.25	2.223	14	294.42	2.993	12

increases when the average passenger waiting time falls within a small range (approximately 2−3 min), thereby indicating that a certain degree of passenger loss rate can obtain relatively high-quality non-inferior solutions. Moreover, the solution with the highest passenger loss rate presents the worst quality solution when the average passenger waiting time is between 4.5 and 6 min, mainly because few people actually take the feeder buses.

First, we compare the MFED and MDEF modes. The result (shown in Fig. 9) demonstrates that the MFED mode outperforms the MDEF mode, thereby indicating that the demand responsive mode is preferred to serve the evening passenger demand. This conclusion is consistent with the finding of Li and Quadrifoglio (2010). As previously mentioned, the arrival of passengers in the morning is unevenly distributed in comparison with passengers
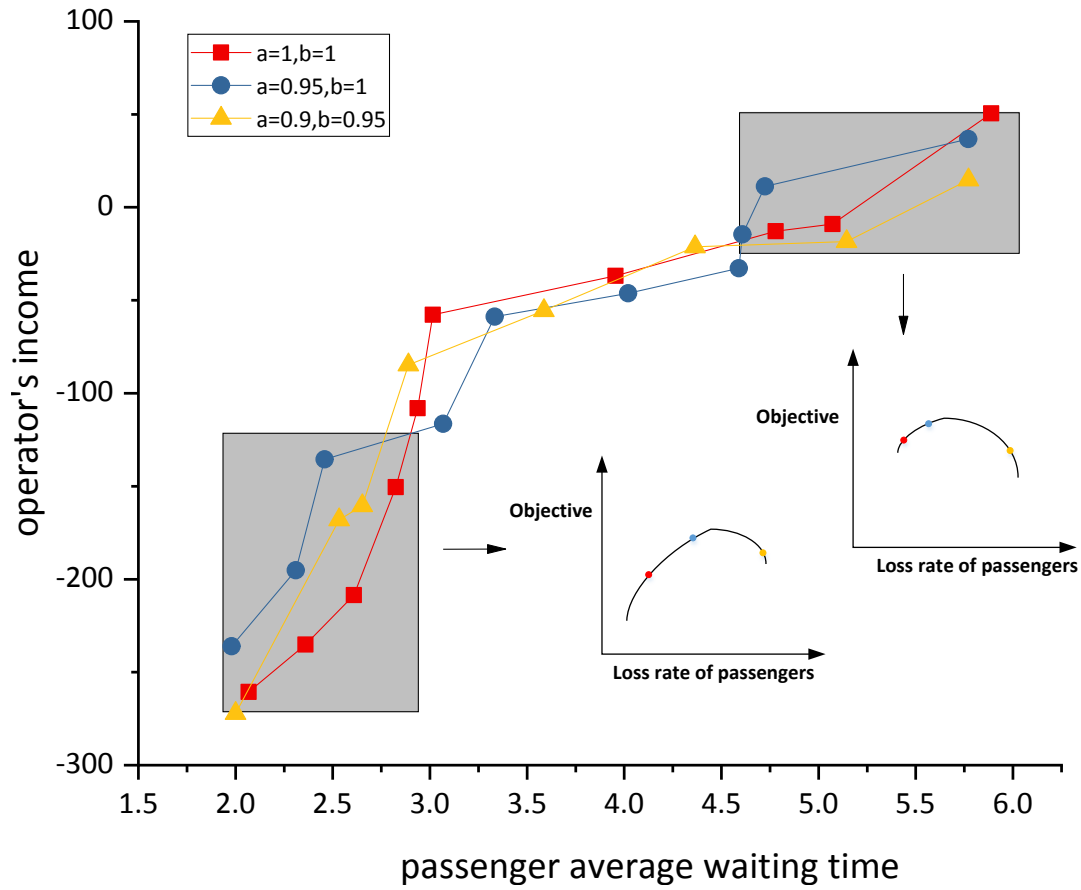
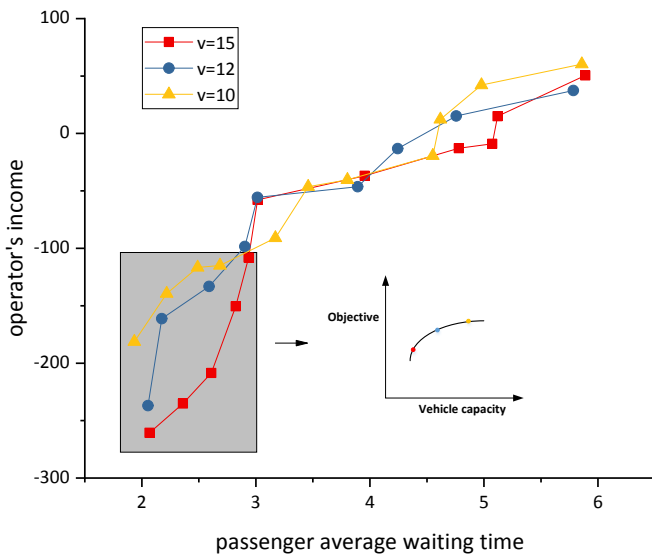Fig. 7. Sensitivity of the objective function to different passenger loss rates.

Fig. 8. Sensitivity of the objective function to different vehicle capacities.
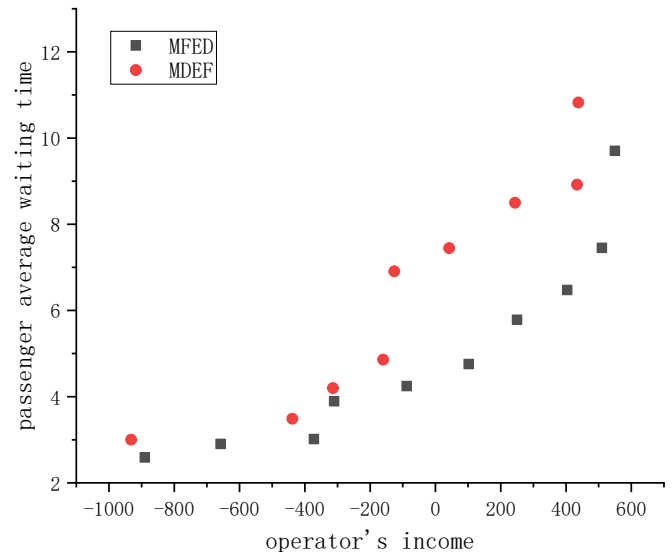

Fig. 9. Non-inferior solutions of the MFED and MDEF modes.

egressing in batches with the same interval (that is, the arrival interval of the train) in the evening. This finding may indicate that the MDEF service (i.e., MDEF mode) will be appropriate. In contrast to our intuitive perception, the results show that the MFED mode outperforms the MDEF mode. This finding indicates that the pattern of fixed frequencies is unsuitable for the even demand

distribution. This conclusion is easy to understand because the fleet size and vehicle schedules for multiple facets must be jointly planned considering the overall profit from the perspectives of passengers and operators. Furthermore, the number of passengers on each route is particularly important. This aspect is related to busload factor and indirectly reflects the requirement to dispatch a

vehicle.

6.3.2. Comparison with the MFEF mode without considering the influence of bike-sharing systems

In practice, shared bikes are unsuitable alternatives for most passengers when the weather is adverse (such as rainy or snowy days), and thus the operator can arrange the feeder bus schedule without considering the effect of bike-sharing. As previously defined, MFEF indicates that each feeder bus line has a fixed schedule during the morning and evening operation periods. In this case, we compare the average passenger waiting time and operational cost under the MFED and MFEF modes. The result is presented in Fig. 10. The number of population and iterations are 100 and 200, correspondingly.

Evidently, the solution quality is generally better under the MFED service than under the MFEF mode. A marginal difference in the operator's cost is observed when the average passenger waiting time is approximately 4 min. However, a distinct difference in the operator's cost is observed when the passengers' average waiting time is approximately 6 min. Furthermore, the increase in the passenger waiting time for the MFED mode results in minimal growth in the operator's profits when the passengers' average waiting time is 8 min. Similarly, this phenomenon applies to the MFEF mode when the passenger's average waiting time is 10 min.

In practice, two thresholds (4 and 8 min for the MFED mode in the abovementioned example) are used for average passenger waiting time, which correspond to two operating schedules. The operating company must select the operating schedules that the relatively low threshold corresponds to when it aims to serve considerable passengers (while ensuring the income as high as possible). By contrast, the operating company must select the operating schedules that the relatively high threshold corresponds to when it aims to gain considerable profits (while ensuring average passenger waiting time as low as possible).

6.3.3. Comparison with the MFEF mode considering the influence of bike-sharing systems

We also compare the average passenger waiting time and operation cost under different modes considering the effects of bike-sharing on feeder buses. The result is illustrated in Fig. 11.

In this figure, the MFED mode generally performs better than the MFEF mode. From the passengers' perspective, the solutions under both modes are relatively close when the average waiting time is approximately 4 min. However, the amount of non-inferior solutions is evidently less under the MFED mode than under the MFEF mode. In this case, a small increase in passengers' average waiting time can result in a large increase in operators' profits. The opposite outcome occurs when the passengers' average waiting time is 7 min. This condition is due to the passenger may switch to shared bikes if the bus waiting time is particularly long. Consequently, the number of served passengers decreases, thus slowly increasing the operational profits.

Fundamentally, the operator can still select the satisfying bus schedules in accordance with valuable factors, such as limited funds and living rhythm, that is, time sensitivity of urban residents, when bike-sharing and feeder transit services are available for passengers to connect the metro line. In addition, the manner by which to determine the optimal fleet size and bus schedules when the metro train is delayed for certain reasons, that is, the train arrival interval is unfixed, will be explored in future research.

7. Conclusions and further research

Few studies considered the bimodal period when optimizing feeder transit service. In this study, we propose a hybrid operation mode, which combines the fixed and dynamic schedules in a bimodal period to connect the metro system. However, when optimizing the dynamic schedules, the fleet size is usually set to be fixed values. We optimized them jointly in this paper under MFED operation modes by creating a multi-objective model, which aims at minimizing the average waiting time of passengers and maximizing the operator's profits. In the modeling process, the effect of bike-sharing on buses is incorporated to mimic the interplay between public transit and biking, which has rarely been addressed in the previous literature.

Moreover, we propose an improved MPSO algorithm to solve the multi-objective model. An empirical study with three residential communities and one metro station in Chengdu, China is adopted,
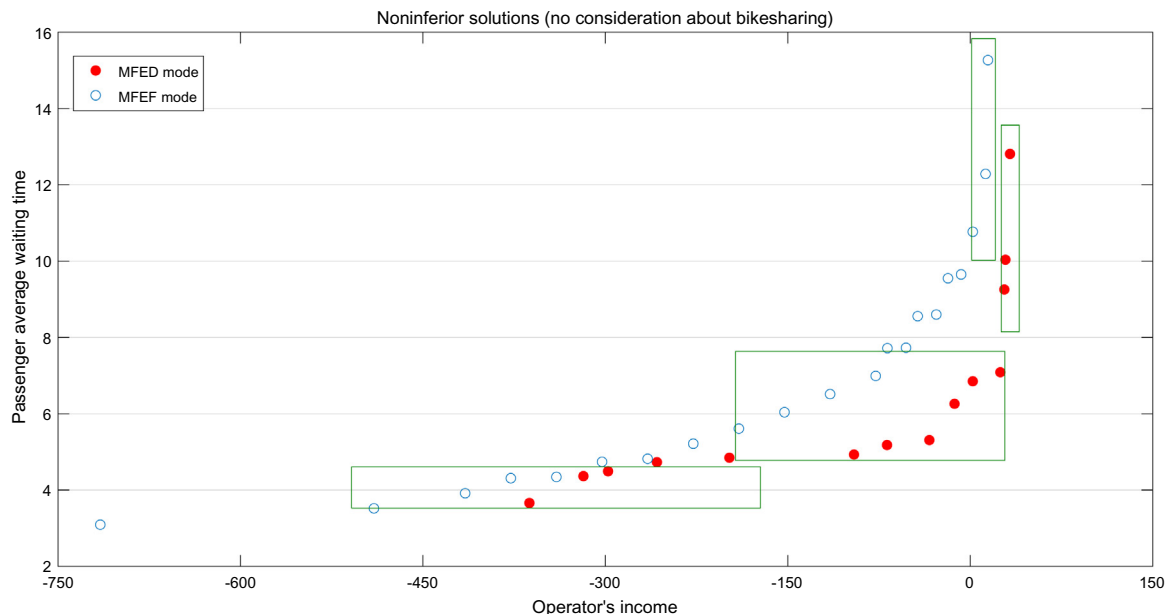

Fig. 10. Non-inferior solutions of the MFED and MFEF modes (regardless of the effect of bike-sharing).
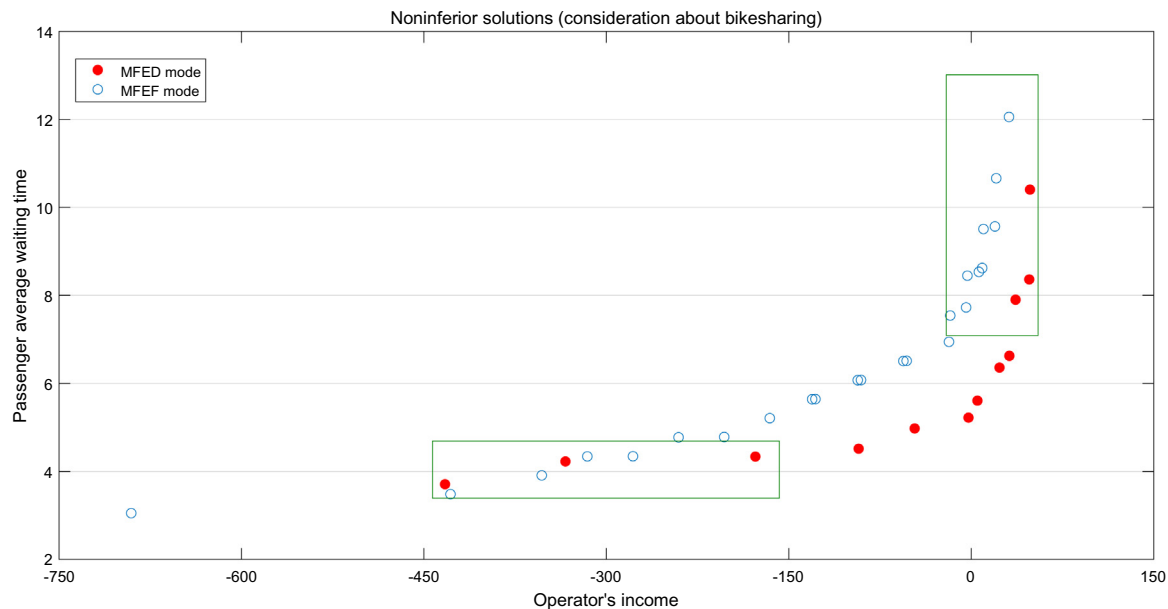
Fig. 11. Non-inferior solutions of the MFED and MFEF modes (considering the effect of bike-sharing).

and the effectiveness and efficiency of the proposed algorithm are tested. No significant difference is observed in the quality of the solutions obtained by the two algorithms by comparing NSGA-II, which has been extensively used to solve the multi-objective problem. However, the computational time is significantly diminished mainly because NSGA-II can search for considerable solution regions. A sensitivity analysis of passenger loss rate during the waiting period and vehicle capacity proves the efficiency and applicability of the MFED mode. In addition, we compare this mode with the other hybrid operation mode (i.e., MDEF) to assist transit authorities in selecting the optimal feeder bus operating policy. The result showed that the MFED operation mode could achieve higher quality solutions than MDEF mode. The findings can assist transit authorities in making optimal feeder bus operating policies to provide high-quality fixed/demand-responsive transit service. Besides, average passenger waiting time can reflect the satisfaction level of passengers regarding transit service to a certain extent, thereby the proposed optimization framework also helps transit authorities to weigh the profits against service quality for decision making. For example, how many buses should be procured in advance to achieve a win-win situation for both passengers and operators.

We further examine the influence of bike-sharing systems by comparing it with the MFEF mode with/without biking. Both scenarios can be applied in practice. For example, transit operators can ignore the effect of bike-sharing under extreme weather conditions and appropriately adjust feeder bus scheduling to obtain a balance between passenger satisfaction level and operating cost. Interestingly, a marginal increase in the waiting time leads to a large gain in operating profits when passengers' average waiting time is relatively short (approximately 4 min in the example). The opposite outcome occurs when the waiting time has reached a certain value (approximately 7 min in the second example). This situation is reasonable because passengers may not opt to take the feeder bus if the waiting time is excessively long, thereby resulting in the loss of travel demand.

For future research, researchers who are interested in this field must attempt to consider the following aspects: (1) The influence of feeder bus fare on passenger's travel choice behavior; (2) stochastic running time must be considered in the optimization model; and (3) unreliable train schedules and probabilistic passengers' walking time can be also examined.

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